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Short Communication

Damage identification using Fourier coefficients of response

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Abstract

This paper proposes a methodology to detect the location and extent of damage in mechanical systems using forced response data. A residual vector defined from vibration response and modeled system matrices forms the objective function of the optimization problem. The residual elements are expressed in terms of unknown stiffness-reduction factors and known Fourier coefficients. Generally, Fourier transform of homogeneous response has peaks at the natural frequencies with amplitudes as real and imaginary coefficients. Stiffness reduction factors are arrived by solving optimization problem using genetic algorithms (GAs) with tournament selection strategy. It is shown that the methodology can be applied to a system of any degree of freedom. Two examples are illustrated to obtain the stiffness reduction factors. The results are shown in the form of tables and graphs.
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1. Introduction

The damage detection is an important requirement for high-speed machines and structures. It is strongly necessary to detect damage at an early stage to avoid catastrophic failure. When there is damage in the form of crack in a rotor, it reduces stiffness of the structure and hence reducing the natural frequencies. Some times the vibration signature can also be used to predict the presence and location of the damage. Rotating machinery produces vibration signatures depending upon the mechanism. A faulty motor, bearing, gearbox, fan and any electrical or mechanical component may produce strong harmonics and side bands, which are to be distinguished from other frequency contents. Thus, vibration signature is sensitive to the type and intensity of damage.

Many methods have been presented for detection of damage in rotors. Detection of crack for non-rotating structure is an ideal problem, where a structure is stopped and checked for damage. Due to rotation, instead of a stationary member the second and higher harmonics of rotational frequency influence the resulting vibration signature. The amplitudes of these harmonics can be measurable only if the frequency of one of the harmonics closely matches any of the natural frequencies. This signature analysis, however predicts the presence of damage, but the location of damage is not an easy task. Many authors studied identification of damage parameters using the response characteristics of a system, in terms of residual vector defined from the model

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and measured data. Ratan et al. [1] employed a residual vector based on frequency response coefficients to determine the location and extent of crack in a rotor system. Here, the non-zero residual elements indicate the crack location. Baruh and Ratan [2] defined the concept of residue and its application for detecting and locating cracks in a truss structure. Mannan and Richardson [3] described the vibration as a sensitive indicator of physical integrity of a mechanical system. Here, the structural fault is detected and located from the changes in measured frequency response function. Sekar et al. [4] employed mechanical impedance approach for detection and monitoring of slant cracks in rotors. Srinivas et al. [5] presented an optimization approach using genetic algorithms (GAs) for obtaining the location and extent of damage using the transient vibration response. Rao et al. [6] illustrated the use of variation in the natural frequencies and mode shapes for detection and location of damage.

Present paper introduces the concept of residual force formed from Fourier coefficients of response and system matrices. To authors knowledge, this has not been attempted in literature. Fourier transform of the homogeneous response of a system is taken. The coefficients are collected at all the nodes for every natural frequency. Natural frequencies and corresponding Fourier coefficients finally form the residual vector in terms of unknown stiffness reduction factors. This vector is minimized for obtaining state of damage in terms of optimal stiffness reduction factors. Thus the approach identifies location and extent of the damage within the system. Methodology is illustrated with two examples: one for a single degree of freedom vehicle model and other for a double disk cracked rotor system. The next section describes the detailed procedure of the residual vector formulation.

2. Mathematical formulation

The equations of the motion of a mechanical system having n degrees of freedom are obtained by assembling the mass, stiffness and damping matrices along with position and force vector associated with every element of the system. It can be written as

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{F}, \tag{1}$$

where $\mathbf{X}(t)$ is displacement vector containing nodal displacements associated with the end points of the elements and $\mathbf{F}$ denotes the vector of external excitations. The vibration response of a system usually contains a homogeneous part and particular integral due to the vector $\mathbf{F}$. In Fourier transform of time–response of a system, the peaks are observed at natural frequencies. By noting the peak values in Fourier Transform of the response it is possible to determine the mode shapes also. The homogeneous response is obtained as a solution of free vibration problem with $\mathbf{F}=0$ in Eq. (1) as follows:

$$\mathbf{X}_h=\sum_{i=1}^{2n}a_i b_i \mathrm{e}^{\lambda_i t}=\sum_{i=1}^{n}\{\alpha_i\mathbf{B}_i\mathrm{e}^{j\lambda_i t}+\bar{\alpha}_i\bar{\mathbf{B}}_i\mathrm{e}^{-j\lambda_i t}\}, \tag{2}$$

where a_i, α_i and $\bar{\alpha}_i$ are constants associated with initial conditions.

The $2n$ complex eigenvalues λ and the corresponding eigenvectors $\mathbf{B}$ ($\mathbf{B}_i$ and its complex conjugate $\bar{\mathbf{B}}_i$) are obtained by solving

$$\{\mathbf{K}-\lambda_i^2\mathbf{M}+j\lambda_i\mathbf{C}\}\mathbf{B}_i=0 \quad \text{where} \quad i=1,2,3,\ldots,n \tag{3a}$$

and

$$\{\mathbf{K}-\lambda_i^2\mathbf{M}-j\lambda_i\mathbf{C}\}\bar{\mathbf{B}}_i=0 \quad \text{where} \quad i=1,2,3,\ldots,n. \tag{3b}$$

That is

$$\left(\begin{bmatrix}0 & \mathbf{I}\\ -\mathbf{K} & 0\end{bmatrix}-\lambda\begin{bmatrix}\mathbf{I} & 0\\ \mathbf{C} & \mathbf{M}\end{bmatrix}\right)\mathbf{B}=0. \tag{4}$$

If eigenvalues are real and positive then eigenvectors can be expressed as $\mathbf{B}_i=(\mathbf{r}_i+j\mathbf{s}_i)$. The real and imaginary portions of $\mathbf{X}$ independently satisfy Eq. (1), as the matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are real. This means the actual response is a linear combination of real and imaginary parts of $\mathbf{X}_h$, namely $(\mathbf{r}_i\cos\lambda_i t-\mathbf{s}_i\sin\lambda_i t)$ and

($\mathbf{r}_i \sin \lambda_i t - \mathbf{s}_i \cos\lambda_i t$). Thus, $\mathbf{X}_h$ can be now written as

$$\mathbf{X}_h = \sum_{i=1}^{n}[\mathbf{c}_i \cos \lambda_i t + \mathbf{d}_i \sin \lambda_i t], \tag{5}$$

where $\mathbf{c}_i = \mathbf{r}_i + \beta_i \mathbf{s}_i$, and $\mathbf{d}_i = \mathbf{r}_i - \beta_i \mathbf{s}_i$ for $i = 1,2,\ldots,n$ with β_i as the constants depending on the initial conditions of the system. Here, $\mathbf{c}_i$ and $\mathbf{d}_i$ are real and imaginary components of vibration amplitudes at natural frequencies in Fourier transform of homogeneous response. Thus $\mathbf{c}_i$ and $\mathbf{d}_i$ are measured from the Fourier transform of homogeneous solution. Now substituting, $\mathbf{X}_h$ in homogeneous equations of motion

$$\sum_{i=1}^{n}[(\mathbf{K} - \lambda_i^2\mathbf{M})c_i + \lambda_i \mathbf{C}\mathbf{d}_i]\cos \lambda_i t + \sum_{i=1}^{n}[(\mathbf{K} - \lambda_i^2\mathbf{M})\mathbf{d}_i - \lambda_i \mathbf{C}\mathbf{c}_i]\sin \lambda_i t = 0. \tag{6}$$

Hence, the coefficients of $\cos \lambda_i t$ and $\sin \lambda_i t$ should separately equal to zero.

Thus, for $i = 1,2,3,\ldots,n$,

$$(\mathbf{K} - \lambda_i^2\mathbf{M})\mathbf{c}_i + \lambda_i \mathbf{C}\mathbf{d}_i = 0, \tag{7a}$$

$$(\mathbf{K} - \lambda_i^2\mathbf{M})\mathbf{d}_i + \lambda_i \mathbf{C}\mathbf{c}_i = 0. \tag{7b}$$

Except at initial stages, the right-hand sides of above equations may not always be zero due to an inherent damage in the system. The non-zero right-hand side elements constitute the residual-vector. Thus, finally for any problem the residual vector is defined simply by considering one of the Eq. (7) as

$$\mathbf{R}_i = (\mathbf{K}_d - \lambda_i^2\mathbf{M})\mathbf{c}_i + \lambda_i \mathbf{C}\mathbf{d}_i, \tag{8}$$

Here, $\mathbf{K}_d$ is the overall damaged stiffness matrix having damage in all or some of m elements, which is described in terms of original elemental stiffness matrices $\mathbf{K}^{(p)}$ and the elemental reduction factors γ_p of element 'p' according to the relation

$$\mathbf{K}_d = \sum_{p=1}^{m} \mathbf{K}^{(p)}\gamma_p. \tag{9}$$

The residual vector is thus defined for each natural frequency, indicating the use of more than one residual vector for correct assessment of damage.

3. Genetic algorithms

Optimization of nonlinear programming problems is a continuously challenging issue. Especially, the problems involving non-unique solutions are seeking the help of non-conventional optimization techniques. GAs represent a popular approach to stochastic optimization, especially as relates to the global optimization problem of finding the best solution among multiple local minima. GAs represent a special case of the more general class of evolutionary computation algorithms (which also includes methods such as evolutionary programming and evolution strategies). The GAs applies when the elements are real-, discrete-, or complex-valued. As indicating from the name, the GAs is based loosely on principles of natural evolution and survival of the fittest. In fact, in GA terminology, an equivalent maximization criterion is often referred to as the fitness function to emphasize the evolutionary concept of the fittest of a species. The steps of a basic form of the GAs are given below. These steps are general enough to govern many implementations of GAs. Performance of a GAs typically depends greatly on the implementation details, just as with other stochastic optimization algorithms.

Step 0 (*initialization*): Randomly generate an initial population of N chromosomes and evaluate the fitness function for each of the chromosomes.

Step 1 (*parent selection*): Set $N_e = 0$, if elitism strategy is not used; $0<N_e<N$ otherwise. Select with replacement ($N-N_e$) parents from the full population. The parents are selected according to their fitness, with those chromosomes having a higher fitness value being selected more often.

Step 2 (*crossover*): For each pair of parents identified in step 1, perform crossover on the parents at a randomly (perhaps uniformly) chosen splice point (or points if using multipoint crossover) with probability p_c. If no crossover takes place, then form two offspring that are exact copies (clones) of the two parents.

Step 3 (*replacement and mutation*): While retaining the N_e best chromosomes from the previous generation, replace the remaining $(N-N_e)$ chromosomes with the current population of offspring from Step 2. For the bit-based implementations, mutate the individual bits with probability p_m; for real coded implementations, use an alternative form of ''small'' modification (in either case, one has the option of choosing whether to make the N_e elitist chromosomes candidates for mutation).

Step 4 (*fitness and end test*): Compute the fitness values for the new population of N chromosomes. Terminate the algorithm if the stopping criterion is met or if the budget of fitness function evaluations is exhausted; else return to Step 1.

In tournament selection approach, '*n*' individuals are selected at random and the fittest is sorted out. Here, the element of population is chosen for passing into next generation if it is better (has better fitness value) than several randomly selected opponents. Tournament size is selection parameter. Tournament selection differs from explicitly ranking schemes in that it doesn't need to sort population during its work. Tournament selection allows parallel execution during choosing members of new generation. It is, just like ranking schemes, invariant to translation (adding same value to the fitness of all item in population) and scaling (multiplying fitness of all items in population with some value). More details of GA can be found in open literature [7]. Fig. 1 shows the flowchart of the present methodology using GA.

4. Results and discussions

The methodology is illustrated with two examples, where the stiffness variation influences the response behavior considerably. The first example is a model of a vehicle traveling over a bumpy road surface as shown in Fig. 2. It is assumed that the vehicle vibrates only in the vertical direction, the stiffness and damping effects of the tire are neglected and tire has good traction and never leaves the road surface.

The equation of motion of this system is

$$m\ddot{y} + C(\dot{y} - \dot{z}) + k(y - z) = 0. \tag{10}$$

Supposing that it travels at a constant speed v, the road roughness can be approximated by a sinusoid defined by

$$z = Z\,\sin\frac{2\pi x}{L} = Z\,\sin\,\omega t, \tag{11}$$

where $\omega = (2\pi v/L)$ is angular velocity of harmonic motion.

The equation of harmonic moving base system now becomes

$$m\ddot{y} + C\dot{y} + ky = C\omega Z\,\cos\,\omega t + kZ\,\sin\,\omega t. \tag{12}$$

The solution contains homogeneous part and particular function due to cosine and sine components. With a simulated vehicle mass of $m = 1000\,\mathrm{kg}$, spring stiffness of $k = 100\,\mathrm{kN\,m^{-1}}$ and a damping ratio of 0.1, the Fourier Transform of response for various conditions of damage is shown in the Fig. 3. The Fourier coefficients corresponding to real and imaginary components c_j and d_j are listed in the Table 1. GAs with a population size of 20, crossover probability of 0.98 and mutation probability of 0.01 is selected with this one-dimensional minimization problem. Residual vector norm is taken as the objective function of test run. The identified values of stiffness reduction factors $\gamma = 0.289$ and 0.701, respectively for the two damaged cases are found to be close to the assumed values for generation of response.

Second example is a simply supported rotating shaft carrying two disks mounted on elastic supports at two ends. In this system, it is required to find the location and extent of damage from the response information and system matrices. Prediction of crack in a rotor is an important topic of research, where the crack opens and closes (breathed) during the rotor rotation. More recently, many authors [8–13] suggested various crack prediction methods. Only the linear open crack model is considered in this paper.

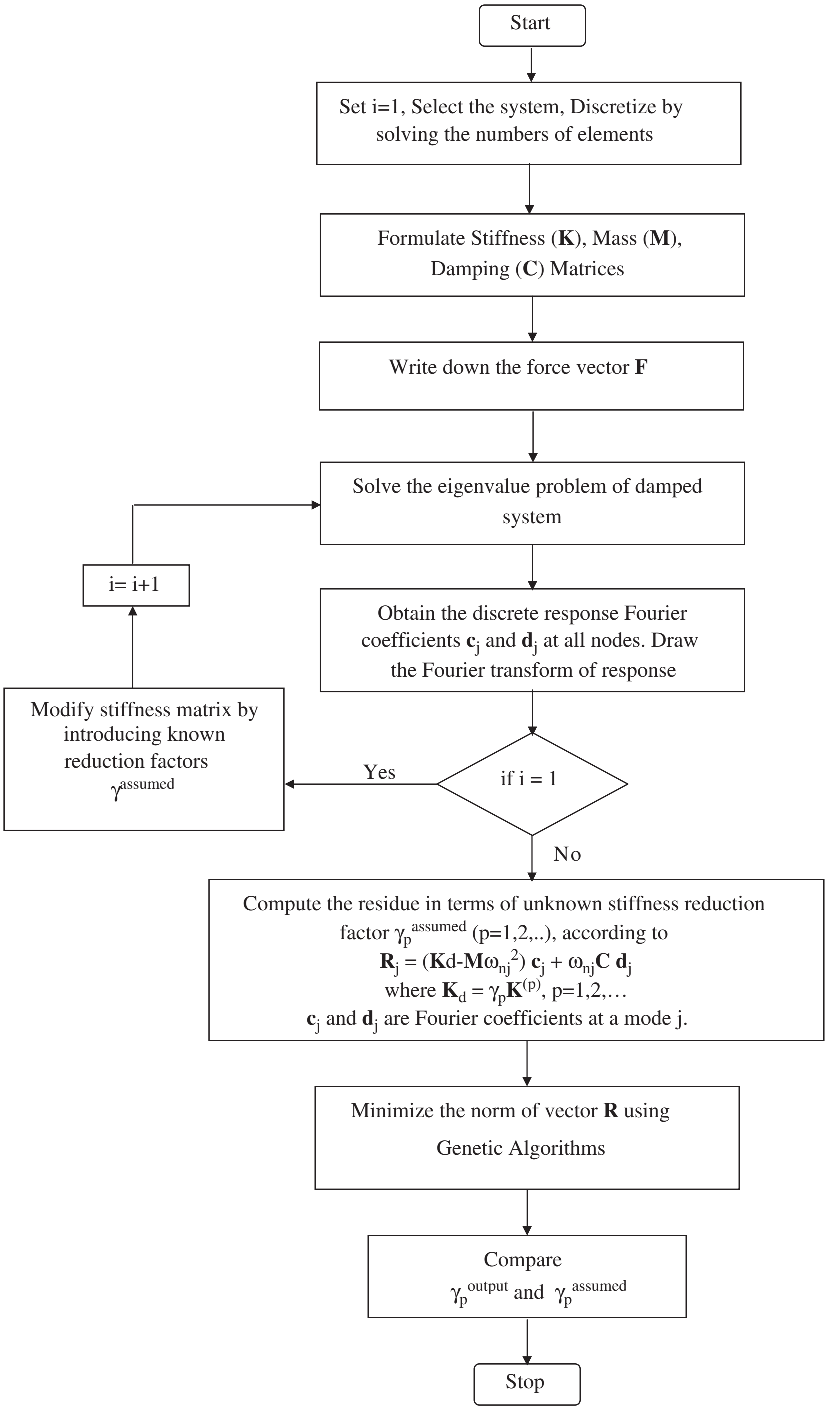

Fig. 1. Detailed flowchart of the approach.

In this formulation, the gyroscopic matrix due to disk rotation is only considered. The entire section is discritized into three unsymmetrical 8-noded beam elements. Element stiffness, mass, gyroscopic matrices are assembled with a code developed in C-language. This program permits to introduce damage in any element in terms of stiffness reductions. Dynamic condensation is employed to reduce size of overall matrices by

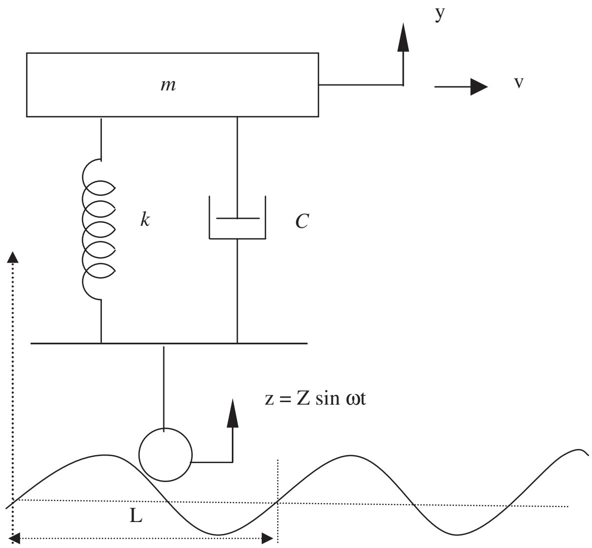

Fig. 2. Vehicle model on a road surface.

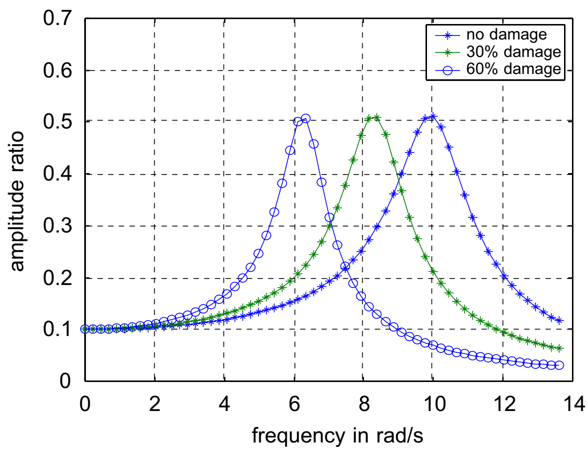

Fig. 3. Fourier transform of the response.

Table 1
Fourier coefficients at a natural frequency (i)

Undamaged system $\omega_n = 9.9499\,\text{rad}\,\text{s}^{-1}$		30% damaged system with $\omega_n = 8.3247\,\text{rad}\,\text{s}^{-1}$		60% damaged system with $\omega_n = 6.2929\,\text{rad}\,\text{s}^{-1}$	
c_i	d_i	c_i	d_i	c_i	d_i
−0.4949	0.0705	−0.5048	0.0719	−0.5147	0.0733

eliminating rotational degrees of freedom. The first-order form of eigenvalue problem is solved to obtain the eigenvalues which are all found to be real. The unbalance response of disk is obtained to check the natural frequencies of the system. This rotating beam carrying disks is a harmonically excited n degree of freedom system. So the excitation vector **F** is given by

$$\mathbf{F} = \mathbf{g}\sin\omega t + \mathbf{h}\cos\omega t, \tag{13}$$

which is usually an imbalance force due to rotor eccentricity. The sub-vectors **g** and **h** are represented by

$$\mathbf{g} = \mathbf{h} = [0 \quad 0 \quad m_1x_1\omega^2 \quad m_1y_1\omega^2 \quad m_2x_2\omega^2 \quad m_2y_2\omega^2 \quad 0 \quad 0]'. \tag{14}$$

Here, m_1 and m_2 are disk masses and (x_1, y_1) and (x_2, y_2) are their coordinates of center of gravity, respectively. The solution can be obtained as a sum of complementary function and particular integral. The particular integral takes the form

$$\mathbf{X}_p = \mathbf{p}\cos\omega t + \mathbf{q}\sin\omega t. \tag{15}$$

Thus, vectors **p** and **q** are obtained by solving

$$\begin{bmatrix} \mathbf{K}-\omega^2\mathbf{M} & \omega\mathbf{C} \\ -\omega\mathbf{C} & \mathbf{K}-\omega^2\mathbf{M} \end{bmatrix} \begin{Bmatrix} \mathbf{p} \\ \mathbf{q} \end{Bmatrix} = \begin{Bmatrix} \mathbf{h} \\ \mathbf{g} \end{Bmatrix}. \tag{16}$$

Here ω is frequency. On the other hand, complementary function depends on arbitrary combination of $2n$ linearly independent functions of the homogeneous problem. Response is obtained in time domain and discrete Fourier transforms are taken over a frequency range to obtain Fourier coefficients at the natural frequencies. Only first natural frequency is considered in the present problem. The three-element rotor system considered in present work is shown in Fig. 4. Properties considered for simulation [1] are shown in Table 2. The material of the rotor shafts has a modulus of elasticity of $E = 2.1\times10^{11}$ Pa and density $\rho = 7806\,\text{kg}\,\text{m}^{-3}$. The natural frequencies of the original system and the system with a small damage introduced in third element of the shaft with a reduction factor $\gamma = 0.4$ are obtained separately and are listed in the Table 3. The Fourier transform of the original response is obtained from MATLAB program and is shown in Fig. 5. It can be seen that the first two modes of the system having peak amplitudes coinciding with those shown in Table 3. The simulated experimental data of a beam with damage in third element is also generated along with undamaged original information. Corresponding Fourier coefficients at first natural frequency along all the nodes are recorded. Residue at the first mode is formulated and objective function is defined in terms of stiffness reduction factors. The problem is solved using GAs with a population size of 40 keeping crossover and

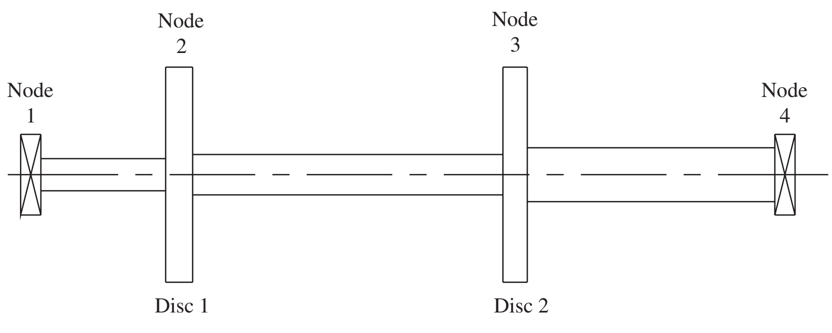

Fig. 4. Three-element rotor system.

Table 2
Properties of rotor system under consideration

Rotor				Disk		
Property	Section-1	Section-2	Section-3	Feature	Disk-1	Disk-2
Diameter (mm)	12	15	20	Mass (m) (kg)	2.06	1.41
Length (mm)	50	120	100	Inertia (I)(kg m^2)	2.91×10^{-3}	1.271×10^{-3}
				Eccentricity (m)	$(-1.2\times10^{-4},0)$	$(6.4\times10^{-4},0)$

Table 3
First four frequencies of 3-element rotor (rad s^{-1})

S. no.	Undamaged	Damaged ($\gamma_3 = 0.4$)
1	784.67	735.18
2	838.77	792.34
3	2435.9	2229.4
4	2452.5	2242.9

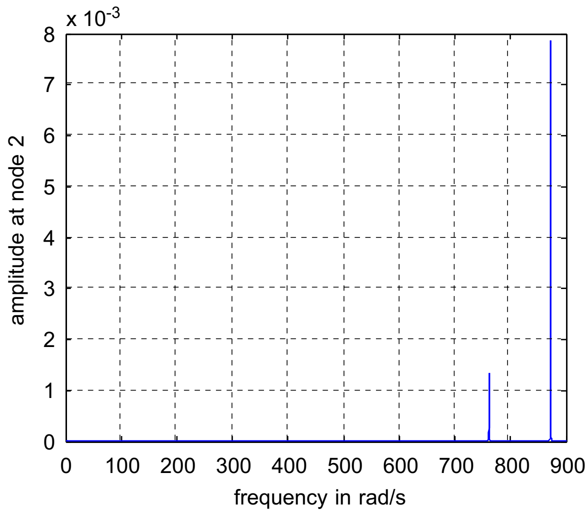

Fig. 5. Unbalance response amplitudes at node-2.

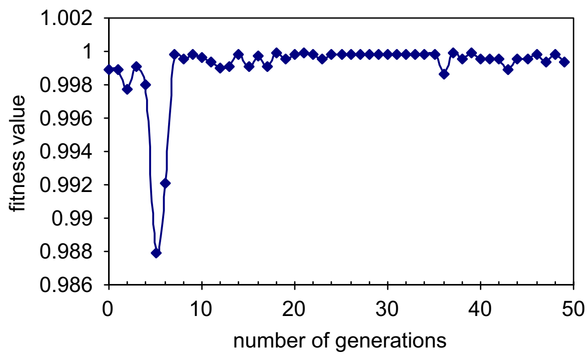

Fig. 6. Training through genetic algorithms.

mutation probabilities the same. Fig. 6 shows the training process in GAs. It took 30 s on a Pentium-IV processor. The final output vector γ is [0.03 0.051 0.389]′.

5. Conclusions

In this paper, an attempt has been made to identify the damage within a system using the available Fourier transform of the response. The approach consists of formulating an objective function in terms of the element stiffness reduction factors using Fourier coefficients at the natural frequencies. Unlike conventional formulation of residue from modal data, the present technique conveniently forms the objective from vector of Fourier coefficients. This novelistic optimization problem was solved through binary coded GAs. Robustness, accuracy and computational time of programs are very encouraging in this work. The methodology can be extended to large-scale problems like vehicular driveline systems.

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